Scribe Notes: Lecture 9 – Localization, Angle, Distance

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Introduction

In this lecture, we discussed wireless localization techniques, focusing on Angle of Arrival (AoA) and Time of Flight (ToF) methods. We explored how these techniques work, their advantages and challenges, especially in multipath environments. We also looked at the "Chronos" approach, which combines phase measurements across multiple frequencies to improve distance estimation.

1 Recap of Angle of Arrival (AoA) Estimation

1.1 Basic Principle

Setup:

- Two antennas, Antenna 1 and Antenna 2, separated by a distance x.
- A signal arrives from a distant source at an angle θ.
- Due to the angle, the signal reaches Antenna 2 slightly before Antenna 1, resulting in a path difference.

Phase Difference Calculation:
The extra distance traveled by the signal to reach Antenna 1 is $x \cos\theta$.

$$
\begin{aligned}
\text{Phase at Antenna 2 } (\phi_2) &= -\frac{2\pi}{\lambda} d, \\
\text{Phase at Antenna 1 } (\phi_1) &= -\frac{2\pi}{\lambda} (d + x \cos\theta), \\
\text{Phase Difference } (\Delta\phi) = \phi_1 - \phi_2 &= -\frac{2\pi}{\lambda} x \cos\theta.
\end{aligned}
$$

Angle Estimation:
By measuring the phase difference ($\Delta\phi$) and knowing x and λ (wavelength), we can solve for θ:

$$\theta = \cos^{-1}\left(-\frac{\Delta\phi\lambda}{2\pi x}\right). \tag{1}$$

1.2 Resolving Ambiguities

Phase Wrapping:

- Phase measurements are modulo 2π, leading to multiple possible angles.
- **Solution:** Limit antenna separation to $x \leq \frac{\lambda}{2}$ to ensure a unique solution for θ.

1.3 Pros and Cons of AoA Estimation

Pros:

- **Higher Accuracy:** Less affected by signal strength variations compared to RSSI-based methods.
- **Uniform Error:** Provides consistent accuracy over different distances.

Cons:

- **Phase Information Required:** Needs precise phase measurements, which may not be readily accessible on commercial devices.
- **Multipath Sensitivity:** Multiple signal paths can lead to incorrect angle estimation.
- **Multiple Access Points Needed:** Typically requires measurements from multiple locations to triangulate the position.

2 Dealing with Multipath in AoA Estimation

2.1 Challenges of Multipath

Multiple Paths:

- Signals can arrive at the receiver via multiple paths due to reflections.
- Each path may have a different angle and delay, complicating the AoA estimation.

2.2 Separating Multiple Paths

Multipath Profile:

- A graph plotting signal power versus angle.
- Peaks in the profile correspond to signals arriving from specific angles.

Objective:
Identify and separate individual paths to determine the direct path from the transmitter.

2.3 Using Antenna Arrays

Extending to Multiple Antennas:

- Using an array of antennas allows for better resolution and the ability to distinguish between multiple paths.
- **Uniform Linear Array (ULA):** Antennas are placed in a straight line with equal spacing x.

Constructing the Multipath Profile:

- **Steering Vector:**

$$P(\theta) = \left| \sum_{i=0}^{N-1} H_i e^{j \frac{2\pi}{\lambda} i x \cos\theta} \right|, \tag{2}$$

where H_i is the channel measurement at antenna i. By adjusting θ and calculating $P(\theta)$, peaks occur at the actual AoAs.

2.4 Resolving Power and Angle Ambiguities

Resolution Improvement:

- Increasing the number of antennas (N) narrows the beamwidth, improving angular resolution.
- **Beamwidth Approximation:**
$$\text{Beamwidth} \approx \frac{\pi}{N}. \tag{3}$$
- Allows separation of closely spaced paths.

2.5 Identifying the Direct Path

Methods:

- **Strongest Path Assumption:** Assume the strongest signal corresponds to the direct path.
  - **Limitation:** May fail if the direct path is obstructed or attenuated.
- **Spatial Consistency:** Use subsets of antennas to create shifted arrays.
  - The direct path remains consistent across shifts, while reflections vary.
- **Combining with Time of Flight:** Use distance measurements to identify the shortest (direct) path.

3 Time of Flight (ToF) for Distance Estimation

3.1 Basic Principle

Concept:
Measure the time it takes for a signal to travel from the transmitter to the receiver.
Distance Calculation:

$$d = c \times t, \tag{4}$$

where c is the speed of light (3×10^8 m/s) and t is the time of flight.

3.2 Challenges with ToF Measurement

Clock Synchronization:

- Transmitter and receiver clocks are not synchronized.
- Nanosecond-level synchronization is required for accurate distance estimation.

Impact of Clock Error:

- Even microsecond-level errors can result in hundreds of meters of distance error.
- For example, a 1 microsecond error corresponds to approximately 300 meters of distance error.

3.3 Mitigating Clock Synchronization Errors

Two-Way Ranging (Round-Trip Time):

1. Device A sends a signal at time T_1.
2. Device B receives it at T_2 and immediately sends a response at T_3.
3. Device A receives the response at T_4.

Time of Flight Calculation:

$$\text{ToF} = \frac{(T_4 - T_1) - (T_3 - T_2)}{2}. \tag{5}$$

- T_1 and T_4: Times recorded by Device A.
- T_2 and T_3: Times recorded by Device B.

Advantages:

- Clock offsets cancel out, eliminating the need for synchronized clocks.
- This method is proposed in fine timing measurement (FTM) protocols for Wi-Fi.

3.4 Limitations of ToF in Multipath Environments

Multipath Interference:

- Reflected signals can cause errors in ToF estimation.
- The measured ToF may correspond to a reflected path rather than the direct path.

Resolution Constraints:

- Limited by signal bandwidth.
- Wider bandwidth allows for finer time resolution.

4 Combining AoA and ToF: The *Chronos* Approach

4.1 Introduction to Chronos

Objective:
Estimate the distance between devices using phase measurements across multiple frequencies. Overcomes the limitations of clock synchronization and multipath interference.

4.2 Phase-Based Distance Estimation

Phase Relation to Distance:
The phase of a received signal is related to the distance d and wavelength λ:

$$\phi = -\frac{2\pi}{\lambda} d \mod 2\pi. \tag{6}$$

Challenge:
Phase measurements are modulo 2π, leading to ambiguity in d.

4.3 Resolving Ambiguity with Multiple Frequencies

Multiple Wavelengths:

By measuring phase at different frequencies (thus different λ), we obtain multiple equations:

$$\phi_i = -\frac{2\pi}{\lambda_i} d \mod 2\pi. \tag{7}$$

Method:

Use the Chinese Remainder Theorem (CRT) to find a common d that satisfies all equations. This reduces ambiguity by aligning the possible distances from each frequency.

Example:

At Frequency f_1:

- Wavelength $\lambda_1 = 5$ cm.
- Measured phase ϕ_1.
- Possible distances: $d = 0.5, 5.5, 10.5, 15.5, \ldots$ cm.

At Frequency f_2:

- Wavelength $\lambda_2 = 7$ cm.
- Measured phase ϕ_2.
- Possible distances: $d = 3.5, 10.5, 17.5, 24.5, \ldots$ cm.

Intersection:

Common distance $d = 10.5$ cm satisfies both measurements.

4.4 Advantages of the Chronos Method

No Need for Synchronized Clocks:

Relies on phase measurements, eliminating clock synchronization issues.

Handles Multipath:

By combining phase measurements across frequencies, the direct path can be isolated.

Compatibility with Existing Hardware:

Utilizes the multiple subcarriers in OFDM systems (e.g., Wi-Fi) to obtain different frequencies.

4.5 Limitations and Practical Considerations

Phase Noise and Measurement Errors:

Requires precise phase measurements, which can be affected by hardware imperfections.

Multipath Complexity:

In environments with severe multipath, additional signal processing is needed to separate paths.

Combining with AoA:

Distance estimation alone is insufficient for localization. Combining distance (from ToF) and angle (from AoA) provides a complete position estimate.

5 Conclusion

Localization in wireless networks can be achieved using AoA and ToF techniques. While each method has its challenges, combining them can enhance accuracy. The Chronos approach offers a practical solution by leveraging phase measurements across multiple frequencies to estimate distance without requiring synchronized clocks, paving the way for high-precision localization applications.